FAQ's

re:

Mean Field Theory
for Chaotic Regime

FAQ's re: Calculations for Chaos

→ what does f mean?

$$\frac{dr}{dz} = \frac{B_r}{B_T}, \quad \frac{r\,d\theta}{dz} = \frac{B_\theta}{B_T} \qquad (B_T \text{ uniform})$$

define Hamiltonian orbits for lines.

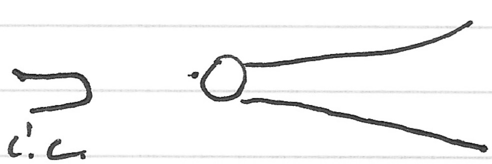
Then f is Liouvillian function for orbits, and $\underline{B} \cdot \underline{\nabla} f = 0$ → Liouville Eqn.

— seek? → $\langle f \rangle$, $\langle f \rangle = \langle f(z, r) \rangle$ <u>mean</u>

begs: scales?
meaning of brackets?

- $\langle f \rangle$ is <u>mean</u> pdf. Idea is to calculate evolution of pdf as lines wander.

i.e.



- implicit is <u>2 scale</u> picture/ansatz.

i.e. $$\tilde{B}_r = \sum_{m,n} B_{m,n}(r) \exp\left[i(m\theta - n\phi)\right]$$

$$\left.\begin{aligned} k_\theta &= m/r \\ k_z &= n/R \end{aligned}\right\} \text{fast scales}$$

$\langle f(z) \rangle \rightarrow$ slow scale

so $$k_z \gg \left| \frac{1}{\langle f \rangle} \frac{\partial \langle f \rangle}{\partial z} \right|$$

and thus:

$$\langle \ \rangle = \int_0^{2\pi} \frac{d\theta}{2\pi} \int_0^{2\pi} \frac{d\phi}{2\pi}$$

$\rightarrow$ evolution equation:

$$\frac{\partial \langle f \rangle}{\partial z} = -\frac{\partial}{\partial r} \left\langle \sum_{m,n} \frac{\tilde{B}_{r,-m,-n}}{B_T} \delta f_{m,n} \right\rangle$$

radial flux of pdf.

$$\equiv -\frac{\partial}{\partial r} \Gamma$$

issues: – hierarchy truncation?
– closure?

→ use linear response iteration
– mean field theory
– quasi-linear theory

why? → $Ku < 1$

$$Ku = \frac{l_{ac}}{\Delta_\perp}\frac{\tilde{B}_r}{B_0}$$

$\Delta_\perp$ →  → radial scale of scattering field

$|\tilde{B}_{r\,m,n}|^2$

$l_{ac} \to \frac{1}{|\Delta k_\parallel|}$ → inverse spatial bandwidth of $k_\parallel$

$Ku < 1$ → weak scattering, many kicks ⇒ diffusion

$Ku > 1$ → strong scattering, QLT not ok.

→ Calculating δf → Linearize!

$$\underline{B}_0 \cdot \nabla \delta f = -\tilde{B}_r \frac{\partial \langle f \rangle}{\partial r}$$

$$\left.\begin{matrix} \delta f \\ \tilde{B} \end{matrix}\right\} \sim e^{i(k_\theta y - k_z z)} \qquad y = r\theta$$

n.b. $k_z \to k_z + i\epsilon$, so $\left\{ \begin{matrix} \delta f \to 0 \text{ as } z \to -\infty \\ \underline{\text{causality}} \end{matrix} \right.$

so
$$\delta f_{\underline{k}} = \frac{i}{\left[k_z - k_y \frac{\langle B_\theta \rangle}{B_T} + i\epsilon \right]} \left(-\frac{\tilde{B}_{r_{\underline{k}}}}{B_0} \frac{\partial \langle f \rangle}{\partial r} \right)$$

$\therefore$

$$\Rightarrow \Gamma = -D_M \frac{\partial \langle f \rangle}{\partial r}$$

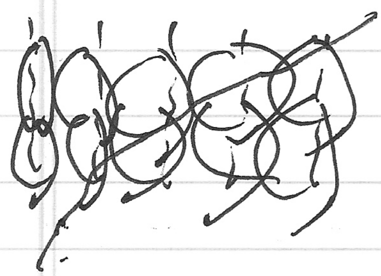
$$\Rightarrow D_M = \sum_{\underline{k}} \left| \frac{\tilde{B}_{r_{\underline{k}}}}{B_0} \right|^2 \pi \delta\left(k_z - k_y \frac{\langle B_\theta \rangle}{B_T} \right)$$

↓

radial diffusion equation for $\langle f \rangle$, due field wandering

→ why D_M ? → trajectory can hop either direction

→ Origin of irreversibility ? → island overlap.



{ Field line scatters/wanders from resonance to resonance

→ D_M → random walk : { step ? time/parallel step increment. ?

$$\delta r \equiv \underset{\text{radial}}{\text{step}} = l_{ac} \frac{\tilde{B}_r}{B_0}$$

parallel step = l_{ac}

$$D \sim l_{ac}^2 \left|\frac{\tilde{B}_r}{B_0}\right|^2 / l_{ac} \sim l_{ac} \left|\frac{\tilde{B}_r}{B_0}\right|^2$$

→ How measure decorrelation length (i.e. length over which orbit scatters from its un-perturbed trajectory) ?

~ FOM

$$\overline{k_\theta^2}\, r^2 \langle \delta\theta^2 \rangle \sim 1$$

$$\Rightarrow \quad m^2 \langle \delta\theta^2 \rangle \sim 1$$

$$\frac{d\theta}{dz} = \frac{\langle B_\theta \rangle}{r B_z} \qquad \langle B_\theta \rangle = \langle B_\theta(r) \rangle$$

so

$$\frac{d}{dz}(\theta_0 + \delta\theta) = \frac{\langle B_\theta(r_0) \rangle}{r_0 B_z} + \delta r \left(\frac{\langle B_\theta \rangle}{r B_z} \right)'_{r_0}$$

$$\delta\theta \sim \int^z dz' \, \frac{\delta r}{R} \left(\frac{1}{q(r)} \right)'$$

$$= \int^z dz' \, \delta r \left(\frac{1}{R q(r)} \right)'$$

$$\langle \delta\theta^2 \rangle \sim D_M z^3 \left[\left(\frac{1}{R q} \right)' \right]^2$$

$$\overline{m^2} \langle \delta\theta^2 \rangle \sim 1 \sim \overline{m^2} D_M z^3 \frac{q'^2}{R^2 (q^2)^2}$$

$$\Rightarrow \ l_c \sim \left[k_\theta^2 D_M \left(\frac{q'^2 r^2}{q^2} \right) \left(\frac{1}{R\varepsilon} \right)^2 \right]^{-1/3}$$

$\rightarrow$ How does l_c relate to Lyapunov exponent

$h \equiv$ rate for exponential divergence of orbits

$l_c \equiv$ rate for exponential divergence from linear orbit.

$\Rightarrow \ h \sim l_c$.